Exam Technique (12 pages; 8/5/24)

Contents

(A) Presentation of work

(1) If a result is to be proved, and if you are writing this result out at the start of the question (though this may not be necessary), make it clear that this is something to be proved (rather than part of your working), so that the marker doesn't have to read over it.

(2) Suppose a candidate has written the following:

$xy < xz$

$y < z$

$x > 0$

As it stands, it is not clear whether $y < z$ is supposed to lead on from $xy < xz$ (which would be incorrect as it stands, as x could be negative), or whether $y < z$ is a result established earlier (or perhaps stated in the question). Likewise, is $x > 0$ being deduced, or brought in from somewhere else?

A revised version of the above might be:

From (*), $xy < xz$

Also, the question states that $y < z$

Hence $x > 0$

(3) Example

Original version

$\frac{dr}{d\theta} = cos\theta - sin\theta$

$\frac{dr}{d\theta} = 0 \Rightarrow tan\theta = 1 \ (cos\theta \neq 0)$

Improved version

$\frac{dr}{d\theta} = cos\theta - sin\theta$ (*)

$\frac{dr}{d\theta} = 0 \Rightarrow \; cos\theta = sin\theta$

$\Rightarrow \frac{sin\theta}{cos\theta} = 1$; ie $tan\theta = 1$, provided $cos\theta \neq 0$

But, from (*), $cos\theta = 0 \Rightarrow sin\theta = 1$, so that $\frac{dr}{d\theta} \neq 0$

So, as we are assuming that $\frac{dr}{d\theta} = 0$, it follows that $cos\theta \neq 0$, and hence $tan\theta = 1$

(4) To avoid any uncertainty, each statement in your working needs something to show where it comes from. Often this will just be an implies sign (⇒) (but see the discussion below about the "if and only if" argument).

Introducing something with the word "consider" is a good way to indicate that you are starting a new line of argument.

(5) For "if and only if" proofs, it may be acceptable to indicate that the line of reasoning is reversible (assuming that this is the case, of course); ie by use of the ⇔ sign at each stage.

When proving that $A = B$, be careful not to adopt the following (incorrect) argument:

"$A = B \Rightarrow \cdots \Rightarrow Y = Z$ (eg $0 = 0$) [something that is clearly true]"

What we want to show instead is that

"$Y = Z$ [which is clearly true] $\Rightarrow \cdots \Rightarrow A = B$

This can be got round by writing

"$Y = Z \; \Leftrightarrow \cdots \Leftrightarrow A = B$

(though it isn't usually thought to be that elegant (especially when ending with $0 = 0$), and is best avoided if possible).

A more acceptable method is to show that $A = A'$ and that

$B = B'$, and then demonstrate that A' can be rearranged into B'; or alternatively show that $A' - B' = 0$

(6) Explanations

It can be a good idea to explain what you are doing, for the marker's benefit. (However, credit can't be given for an explanation of what you would do, if you had more time.)

(7) Reasons for showing plenty of working:

(a) It enables you to easily check over your work (as you complete each line).

(b) The marker is kept happy, by not forcing them to do algebra in their head.

(c) If you make a slip, then your method may still be clear (bearing in mind that method marks are usually available).

(d) Full marks may not be awarded in the case of a 'show that' result if there is a jump to the result.

(8) Rounding

Unless stated otherwise, 3 significant figures should be sufficient. Angles in degrees though are usually wanted to 1 dp.

Ensure that sufficient decimal places are retained during the calculation, to avoid 'premature rounding' errors. Obviously the size of the numbers is critical, but usually 5dp would be sufficient.

Recommended format of answer:

$x = 12.34567$

$= 12.3$ (3sf)

[12.34567 would then be used in any subsequent calculations]

(9) Drawing / sketching

The instruction to 'draw' means that graph paper should be used (and points are often to be plotted). The more usual instruction to 'sketch' means that the candidate is expected to use plain paper.

Although the use of graph paper will not be penalised in itself, there is the danger that it will incorrectly show a graph as failing to pass through a particular point on the grid (eg the graph of $y = x^2$ may appear not to pass through the point (1,1)), or passing through a point that it shouldn't do).

Most exam boards scan scripts nowadays. Unfortunately, the scanning devices are almost too powerful, and an erased item can sometimes appear just as clearly on the marker's computer as its replacement. So either cross out and re-do the graph, or make the correction clear by some extra labelling (eg "this is the correct curve").

(B) Checking

(1) Read over each line before moving on to the next one. This is the most efficient way of picking up any errors.

(2) Just before embarking on a solution, re-read the question. Also re-read it when you think you have finished answering the

question, in case there is an additional task that you have forgotten about (or the answer may be required to eg 3sf).

(3) It is also a good idea to re-read the question if you find yourself getting bogged down in awkward algebra, or if you don't seem to be getting anywhere.

(4) Look for ways of checking your answer, or part of your working:

(a) if simultaneous equations have been solved, the solutions may be substituted back into the equations

(b) there may be an alternative approach to (for example) an arithmetic calculation

(c) a reasonableness check (including a possible estimate of a numerical solution)

(C) Problem-solving techniques

(C.1) Generating Ideas

(1) Look out for standard prompts to use a particular result. For example, a reference to a tangent to a circle can suggest the result that the radius and the tangent are perpendicular.

(2) Look ahead in the question, to get on the question-setter's wavelength.

(3) The usual convention is for question parts labelled (i), (ii), (iii) … to be related, whilst question parts labelled (a), (b), (c) …

do not lead on to each other. If the first part of a question seems very easy, it is highly likely to be needed for the next part.

(4) Clues in the question:

(a) If you are told that $x \neq a$, then the solution may well involve a division by $x - a$.

(b) A condition in the form of an inequality may suggest the use of $b^2 - 4ac$ (especially if it involves a squared quantity).

(c) The presence of a $\pm$ sign may suggest that a square root is being taken at some stage.

(d) There may be a clue in another part of the question.

For example, if part (i) involves 2^{2x-x^2} , the presence of $2^{-(x-c)^2}$ in part (iv) suggests that completing the square may help.

(5) Ensure that all of the information provided in the question has been used.

(6) If a topic looks unfamiliar, remember that knowledge outside the syllabus is not assumed, so the question should be self-contained and include definitions of new concepts. Usually such questions turn out to be easier than normal, as the candidate is being rewarded for coping with an unfamiliar topic. Typically the first part will turn out to be quite simple.

(7) The last part of a question won't necessarily be any harder than the earlier parts - especially once you have got on the

question-setter's wavelength. Also, the last part might simply be the final (easy) stage in establishing an interesting result.

(8) Comments

For "comment on ..." questions (especially Statistics), any points made should always relate to specific theoretical ideas (eg when comparing distributions in Statistics, make one point about a measure of average, one point about spread, and perhaps one point about skewness; rather than several points about spread, for example).

Sometimes, a mark may be awarded for "any sensible comment", with a second mark awarded for a justification.

(C.2) Creating equations

The general strategy for many questions is to convert the given information into one or more equations, which then need to be solved. This is especially true of Mechanics questions. Equations can be created from:

(a) information in the question

(b) relevant definitions and theorems

If necessary, create your own variables (for example, a particular length in a diagram).

Sometimes the advantage of creating an equation is that it gives you something to manipulate; ie in order to make further progress.

(C.3) Case by case approach

Example: Solve $\frac{x^2+1}{x^2-1} < 1$

Case 1: $x^2 - 1 < 0$; Case 2: $x^2 - 1 > 0$

(Once we know whether $x^2 - 1$ is positive or negative, we can multiply both sides of the inequality by it.)

(C.4) Reformulating the problem

For example, in order to solve the equation $f(x) = k$, consider where the graph of $y = f(x)$ crosses the line $y = k$.

Or, more generally, rearrange an equation to $f(x) = g(x)$, and find where the curves $y = f(x)$ & $y = g(x)$ meet (or show that they won't meet).

(C.5) Experimenting

Examples:

(i) Draw a diagram.

This may reveal a hidden feature of the problem (eg a triangle may turn out to be right-angled).

(ii) Try out particular values.

Again, this may reveal a hidden feature of the problem (eg if an integer n is involved, then perhaps it has to be even).

(iii) Consider a simpler version of the problem (eg experiment with a simple function such as $y = x^2$).

(D) Algebra

(1) Avoid using the multiplication sign (due to possible confusion with x). A dot can be used to indicate multiplication, but brackets are best if a dot could be confused with a decimal point.

Examples

(a) Write $2xy$, rather than $2 \times x \times y$

(b) You could write $\frac{a}{b} . \frac{c}{d}$ or $(\frac{a}{b})(\frac{c}{d})$

(c) Write $2(3x)$, rather than $2.3x$

(2) Use fractions in working, rather than decimals (fractions being easier to manipulate).

Usually a fractional final answer is acceptable (and definitely better than a rounded decimal, unless this is asked for, or if the question involves lengths or speeds etc - when a decimal answer to 3 sf would usually be expected).

In any event, delay converting from fractions to decimals until the last moment.

Also, at A Level it is quite acceptable to give answers as improper fractions (eg $\frac{5}{4}$). Avoid mixed numbers (eg $1\frac{1}{4}$).

If an 'exact' answer is asked for, this means a fraction (or a multiple of π, or a surd - such as $2 + \sqrt{3}$), rather than a rounded decimal.

If the value of a variable is known (eg $g = 9.8$), avoid substituting this value until the last moment. It makes it easier for you - and the marker - to check your work.

(3) When simplifying an algebraic expression consisting of two or more terms:

(i) Factorise whenever possible

(ii) Don't leave fractions inside brackets

(iii) Only expand brackets when there is nothing else to be done

Example: $\frac{1}{2}n(n+1)+\frac{1}{6}n(n+1)(2n+1)+\frac{1}{4}n^2(n+1)^2$

$=\frac{1}{12}n(n+1)\{6+2(2n+1)+3n(n+1)\}$

$=\frac{1}{12}n(n+1)(3n^2+7n+8)$

(4) Provided the number of equations equals the number of unknowns, a unique solution may be possible. However, if only a ratio of two variables is required, one less equation is needed. Also, the method of equating coefficients provides a way of obtaining more than one piece of information from a single equation.

In vector questions, each equation will represent 2 or 3

components (similarly for complex numbers), so that it may be worthwhile creating a new unknown if it means generating 2 or 3 equations.

eg if the vectors $\underline{u}$ & $\underline{v}$ are parallel, we can write $\underline{u} = k\ \underline{v}$

(5) Use letters to represent recurring expressions.

(E) Common Pitfalls

(1) Not using a specified method. Even if a superior method is used instead, zero marks are usually awarded for not using the method requested in the question.

(2) Using ⇒ when ⇔ is required. (See (A)(4) for "if and only if" proofs.)

(3) Losing a solution of an equation by dividing out a factor.

(4) Multiplying an inequality by a quantity without realising that it is (or could be) negative (eg log(0.5)).

(F) Use of time

You might like to save a relatively straightforward task to complete in the last few minutes of the exam, rather than frantically looking through the paper for something to check.