

## Friction (5 pages; 1/6/24)

(1) Friction between two surfaces acts to oppose relative motion, or attempted relative motion, between the surfaces.

(2) When an object is stationary, the frictional force will match any applied force (along the surface), up to a certain point. For a particular object on a particular surface this point is determined by the coefficient of friction,  $\mu$ .

According to Coulomb's experimental law,  $F \leq \mu R$ ,

where  $F$  is the frictional force, and  $R$  is the normal (ie perpendicular) reaction of the surface on the object.

Thus, when a force  $X \leq \mu R$  is applied to a stationary object,

$F = X$ , and the object remains stationary. This situation is often referred to as 'static' friction.

(3) When  $X > \mu R$ , then the object will move, and the situation becomes one of 'dynamic' (also known as 'kinetic' or 'sliding') friction.

The coefficient of dynamic friction is often significantly lower than that of static friction, and the symbols  $\mu_s$  and  $\mu_d$  are often used.

Static friction then occurs when  $X \leq \mu_s R$ ,

and dynamic friction when  $X > \mu_s R$  (sic), in which case  $F = \mu_d R$ , and by N2L:  $X - \mu_d R = ma$ ,

where  $m$  is the mass of the object, and  $a$  is its acceleration.

## Notes

(i)  $\mu_d$  has been found to be reasonably independent of the relative velocity of the two surfaces.

(ii) Exercises are often simplified by assuming that  $\mu_s$  and  $\mu_d$  have the same value (in which case the symbol  $\mu$  is used throughout).

(iii) Most exercises will concern objects that are either “on the point of sliding” or already sliding, so that the frictional force is then known to be  $\mu R$  (or  $\mu_s R / \mu_d R$ ).

(iv) It has been found that the frictional force doesn't depend on the area of contact between the two surfaces.

(v) Typical values of  $\mu$

steel on steel:  $\mu_s = 0.7, \mu_d = 0.6$

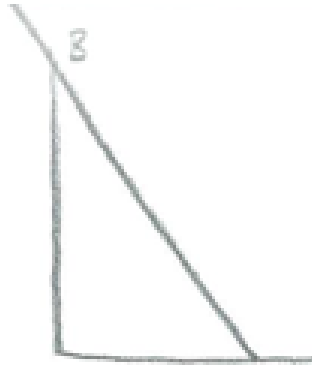
wood on wood:  $\mu_s = 0.4, \mu_d = 0.2$

ice on ice:  $\mu_s = 0.1, \mu_d = 0.03$

(4) If a force is applied to an object which remains at rest on a slope, then the frictional force could be acting up or down the slope. For example, the applied force may be just sufficient to stop the object from sliding down the slope, in which case friction will be opposing the attempted motion, which is down the slope, so that the frictional force acts up the slope - aiding the applied force. If on the other hand the applied force is not quite enough to move the object up the slope, then the attempted motion is up the slope, and the frictional force will act down the slope - countering the applied force.

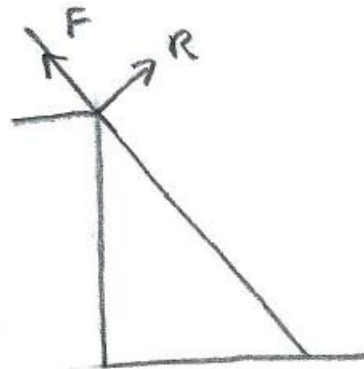
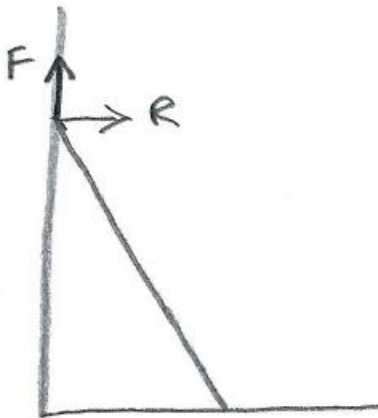
(5) Considering a block on a horizontal surface, or a ladder resting against a wall: the reaction force of the surface on the block or ladder can be treated as having two components: the normal reaction and a frictional force (if applicable).

Exercise: Indicate the forces acting at A and B below (assuming the wall is a rough surface).



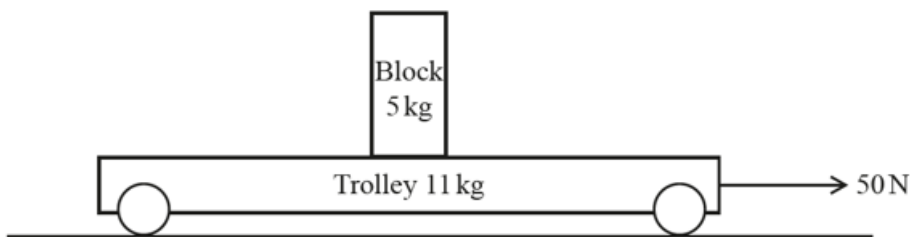
### Solution

Consider the direction in which the ladder would slide, were the wall to be smooth.



## (6) Example

A trolley of mass  $11\text{kg}$  is pulled by a  $50\text{N}$  force along a smooth surface. A block of  $5\text{kg}$  is on top of the trolley, and the coefficient of friction between the block and the trolley is  $0.3$  (which can be taken to apply to both static and dynamic friction). Show that the block slides on the trolley, and find the accelerations of the trolley and the block relative to the surface.

**Solution**

Suppose that the block doesn't slide.

Then N2L for the trolley & block together gives:

$$50 = (11 + 5)a,$$

where  $a$  is the acceleration (to the right) of both the trolley and the block (relative to the surface).

The only force on the block is the frictional force,  $f$  say.

Then N2L for the block gives:

$$f = 5a = 5 \left( \frac{50}{16} \right) = 15.625$$

Now, the maximum possible value for  $f$

$$\text{is } \mu R' = (0.3)(5g) = 14.7$$

So the block must slide, and clearly this will be to the left, relative to the trolley; thus the frictional force acts to the right, opposing the motion of the block relative to the trolley.

Let the accelerations of the block & trolley (relative to the surface) be  $a'$  &  $a$  (to the right).

The frictional force will be the limiting value of 14.7, as above.

Then, applying N2L to the block & trolley separately:

$14.7 = 5a'$  and  $50 - 14.7 = 11a$  (by N3L, there will be a force of 14.7 to the left on the trolley).

Then  $a' = \frac{14.7}{5} = 2.94 \text{ ms}^{-2}$  and  $a = \frac{50-14.7}{11} = 3.21 \text{ ms}^{-2}$  (both to 3sf).

(7) The treatment of friction is very different for the case of a rolling wheel - see separate note.