Mechanics - Important Ideas: Forces (16 pages; 23/5/24)

[See separate note for Friction.]

Contents

(A) Newton's 1st law

This says that an object continues in motion (in a straight line) with a constant speed (which may be zero), unless acted on by an external force. (It isn't usually needed when answering Mechanics questions.)

(B) Newton's 2nd law

This says that $F = ma$, where F is the net force on an object of mass m, and a is the acceleration of the object.

It can be considered to be a vector equation, but the simplest approach is usually to resolve the forces in two convenient perpendicular directions, and set up two $F = ma$ equations.

Note

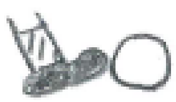
As F may have been defined to be one of several forces acting on the object (friction, for example), it is generally best to write

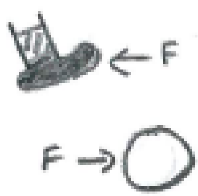
$'F = ma'$. Alternatively $N2L$ can be referred to.

(C) Newton's 3rd Law

This says that "if object A exerts a force on object B, then B exerts an equal and opposite (reaction) force on A".

Example: Football being kicked





If a football is kicked, then we know from experience that the ball exerts a force on the player's foot. Note though that, despite the apparent symmetry, it is the foot that is the cause of this force.

In this example, the force and reaction force both depend on the nature of the objects in question. Thus a balloon would offer minimal resistance to a football boot, and so the boot would only be able to exert a small force on the balloon.

Don't confuse Newton's 3rd law with equilibrium: In the case of equilibrium, the equal forces being considered act on the same object, whereas in the case of Newton's 3rd law the equal forces act on different objects (in this example, one acts on the ball and the other acts on the foot).

(D) Force Diagrams

The first step in many Mechanics problems is to create one or more force diagrams. To avoid any confusion, each diagram should show only the forces on the object under consideration (textbooks don't always do this). If not provided in the question, it may be worth having a separate diagram showing the overall situation - including lengths and angles, but omitting forces.

Each force diagram will enable Newton's 2nd law to be applied, and so generate an equation, which will generally enable one unknown variable to be determined.

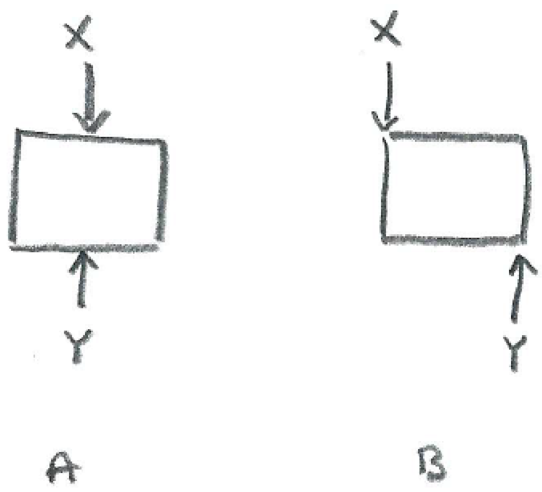
(E) Reaction force between an object and a surface

Consider an object of mass m resting on a horizontal surface. It is tempting to just say that the surface experiences a force of mg due to the weight of the object. The correct reasoning is as follows:

Drawing a force diagram for the object, and applying Newton's 2nd law gives: $R = mg$. Then, by Newton's 3rd law, the reaction force on the surface from the object is R, which we have found to equal mg.

(See the "Man in a lift" example below, for a more complicated situation. Also, the note "Important Ideas - Equilibrium" discusses 2 dimensional situations.)

(F) Particle model



If the forces applied to an object are not liable to cause it to rotate (as in diagram A), then the object may usefully be modelled as a particle (even if it is quite large). The motion of the object is then determined by applying Newton's 2nd law (in general, in two perpendicular directions).

However, if rotation (or potential rotation) is an issue (as in diagram B), then the theory of moments comes into play. The examples in this note assume that a particle model is appropriate (see the note " Important Ideas - Equilibrium" for non-particle ('rigid body') models).

Where more than one particle is involved (as in most of the following examples), the situation is often referred to as one of "connected particles" (even the man in a lift!)

(G) Car pulling a trailer

(G.1) Example

Suppose that a car of mass $2000kg$ (2 tonnes) is pulling a trailer of mass $500kg$. The engine of the car supplies a driving force of $2000N$, and there is air resistance of $600N$ and $400N$ on the car and trailer, respectively. Find the acceleration of the car and trailer, and the tension in the towbar.

Solution

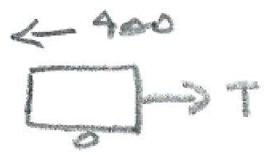
Considering the car and trailer (and towbar) as a single object, a force diagram can be created, and N2L applied:

$2000 - 600 - 400 = (2000 + 500)a$

$\Rightarrow a = \frac{1000}{2500} = 0.4\ ms^{-2}$

[Although it involves mixing units: a mild acceleration for a car travelling on the motorway at 50 mph [miles per hour] would be 1 mph per second. Also, the speed of an olympic sprinter (covering 100m in 10 seconds) is 10 ms^{-1}, which is approximately $\frac{10\times3600}{1600} = 22.5\ mph$. So 1 mph is approximately $\frac{10}{22.5} = 0.44\ ms^{-1}$, and 1 mph per second is approximately 0.44 ms^{-2}.]

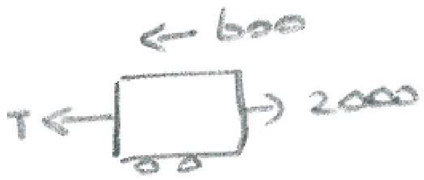
Now drawing a force diagram for the trailer:



$T - 400 = 500(0.4)$, so that $T = 600N$

[The tension in the towbar will be discussed shortly.]

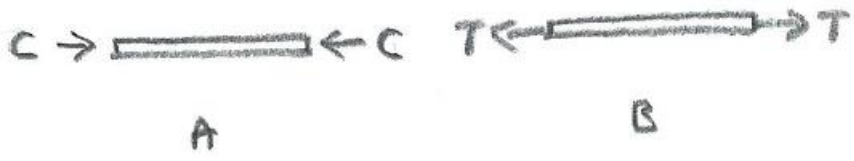
[As a check, we could draw a force diagram for the car:



$2000 - 600 - T = 2000(0.4)$, giving $T = 600N$ again.]

(G.2) Tension in the towbar

Consider a force diagram for the towbar. There are two possibilities: the towbar may be under compression (diagram A), or under tension (diagram B).



Tension will be the more usual situation, with compression generally only occurring in the case of heavy braking (as seen below). For this reason, the symbol T is invariably used - on the understanding that it may turn out to be negative. (Then, for example, a tension of $-500\ N$ would be described as a compression of $500\ N$.)

The assumption that the towbar is subjected to the same force at each end can be justified as follows:

Suppose that the towbar (under tension) is subject to forces T_1 and T_2 at its two ends, as shown in the diagram.

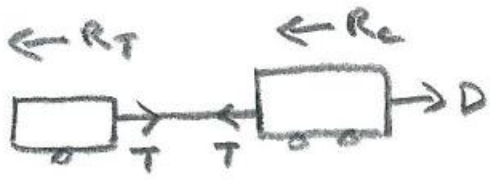
Applying N2L: $T_1 - T_2 = ma$,

where m is the mass of the towbar, and a is the acceleration of the car, trailer and towbar.

If the towbar is assumed to be of negligible mass (or 'light'), so that $m \approx 0$, then $T_1 \approx T_2$.

(G.3) Textbook diagrams

Textbooks will sometimes (confusingly) include all the forces on a single diagram, and draw the arrows on the towbar pointing away from the car and the trailer. These are intended to be forces on the car and trailer (rather than on the towbar) - see the diagram below.



[To be avoided - some forces are on the car, and some are on the trailer.]

(G.4) Compression in the towbar

When the car and trailer are decelerating, we can show that the towbar will be under compression if there is heavy braking, or if the resistance on the trailer is sufficiently small:

If the car and trailer are decelerating at a rate d, then the tension T in the towbar is given by:

$R_T - T = m_T d$,

where m_T is the mass of the trailer, and R_T is the resistance on it.

Thus the towbar will be under compression if

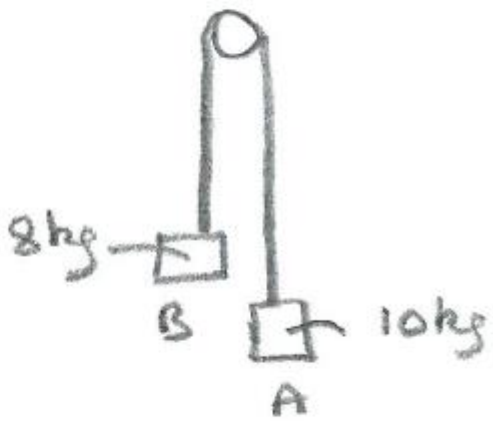
$T = R_T - m_T d < 0$; ie if $R_T < m_T d$;

ie if either the deceleration is big enough (due to heavy braking), or if R_T is sufficiently small.

(For large R_T, the trailer would decelerate at a rate greater than d if it were uncoupled from the towbar, and the tension is needed in order to obtain the smaller deceleration of d.)

(H) Pulley system

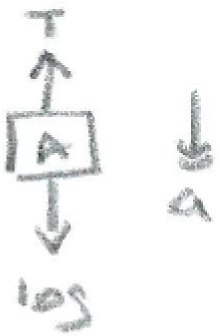
Example



In the case of two blocks on either side of a pulley (as in the diagram), we cannot treat the two blocks and the connecting rope as a single object, as they are not moving in the same direction.

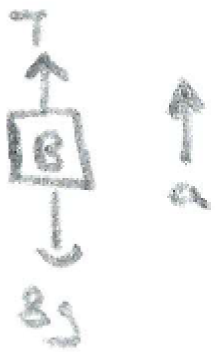
However, we can combine the equations of motion for the two blocks as follows:

Block A



$10g - T = 10a$

Block B



$T - 8g = 8a$

Adding these two eq'ns gives $2g = 18a$, so that $a = \frac{g}{9}\ ms^{-2}$

[This is a fairly typical fraction of g to have as an acceleration, and this fact can be used as a check. Compare $\frac{g}{9} = 1.09\ ms^{-2}$ with the acceleration of the car which was $0.4\ ms^{-2}$.]

Notes

(i) The pulley will usually be described as 'smooth', to indicate that there is no frictional force acting on the connecting rope. This will ensure that the tension is the same at both ends of the rope. This can be demonstrated as follows:

In general, a rope (of mass m) over a pulley has 3 external forces on it: the tension at the two ends (T_1 & T_2) and the frictional force (F) due to the pulley.

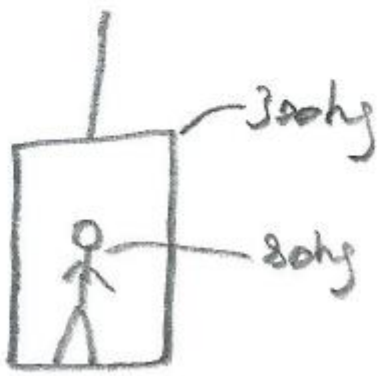
Applying Newton's 2nd law to the rope,

$T_1 - T_2 - F = ma$ (where a is the acceleration of the rope)

If the rope has negligible mass, and if F is also negligible, then $T_1 \approx T_2$.

(ii) The rope will usually be described as inextensible, in order to ensure that all components of the system have the same acceleration.

(I) Man in a lift



Example 1: The lift is accelerating upwards at 0.5 ms^{-2}

Find the tension in the lift cable and the reaction of the man on the floor of the lift.

Solution

Treating the lift and the man as a single object, we can draw a force diagram and apply N2L:

$T - (300 + 80)g = (300 + 80)(0.5)$

$\Rightarrow T = 380(10.3) = 3914\ N$

Then drawing a force diagram for the man:

$R - 80g = 80(0.5) \Rightarrow R = 80(10.3) = 824\ N$

This is the reaction of the floor of the lift on the man, but by N3L the reaction of the man on the floor of the lift is also $824\ N$.

Notes

(i) For exam purposes, any symbols used in the force diagrams should be defined.

(ii) The reaction of the floor of the lift on the man is what the man perceives to be his weight. Compare this with his weight when not accelerating with the lift: $80g = 784\ N$. Thus, when the lift is accelerating upwards, the man feels heavier, compared to his natural weight.

Example 2: As before, but with the lift accelerating downwards at $0.5\ ms^{-2}$

Solution

For the lift and the man: $T - (300 + 80)g = (300 + 80)(-0.5)$

$\Rightarrow T = 380(9.3) = 3534\ N$

For the man: $R - 80g = 80(-0.5) \Rightarrow R = 80(9.3) = 744\ N$

Note

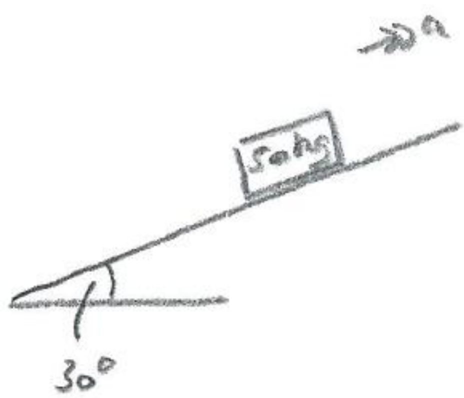
Thus, when the lift is accelerating downwards, the man feels lighter, compared to his natural weight. In the hypothetical case of the lift cable breaking, so that $a = g$, R becomes zero (ie the man experiences weightlessness).

(J) Block on a slope

Example

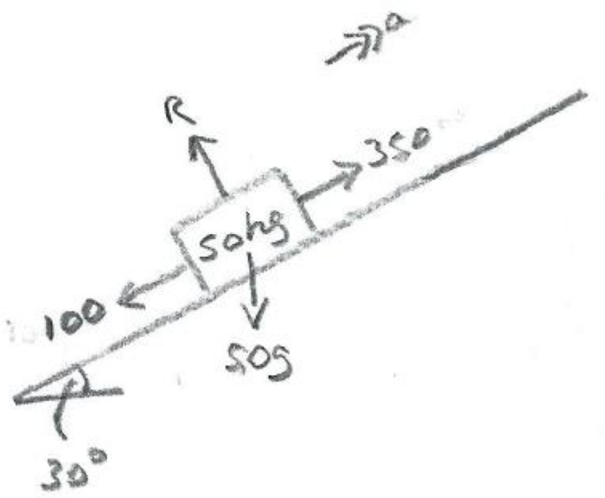
A block of mass $50kg$ is pulled up a slope (which makes an angle of 30° with the horizontal) by a force of $350\ N$, and there is a frictional force of $100\ N$ acting down the slope. Find the acceleration of the block, and the reaction of the slope on the block.

Solution



This is a two-dimensional situation, and we need to resolve the various forces in two perpendicular directions. For many situations, the most convenient directions to choose are vertically and horizontal. However, in this case the best directions will be along and perpendicular to the slope. This takes advantage of the fact that there will be no acceleration perpendicular to the slope - and this will be true whether or not the block is accelerating. (If the block is either stationary or not accelerating, then forces could in theory be resolved vertically and horizontally, but this is not usually done.)

We can draw a force diagram for the block:



Resolving parallel to the slope and applying N2L:

$350 - 100 - 50gsin30° = 50a$

$\Rightarrow a = \frac{1}{50}(250 - 245) = 0.1\ ms^{-2}$

Resolving perpendicular to the slope and applying N2L:

$R - 50gcos30° = 0$

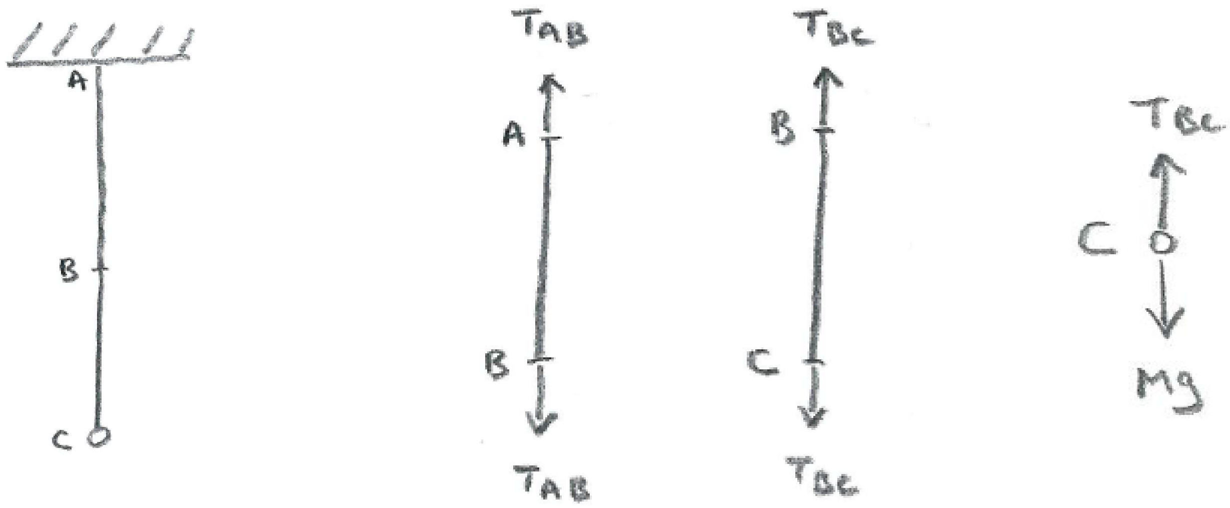
$\Rightarrow R = 424\ N$ (3sf)

(K) Hooke's Law

(1) Consider two elastic strings, AB and BC, as shown below, with an object of mass M at C. The system is in equilibrium.

The two strings have original lengths of l_1 and l_2, respectively; extensions of x_1 and x_2, and moduli of elasticity λ_1 and λ_2.

Force diagrams can be drawn for the strings AB and BC, as well as for the object C (these are also shown below).



AB is in tension, and the forces at each end can be taken to be the same, because the string will be assumed to have negligible mass. To see this, suppose that the forces are T_A and T_B. Then, by N2L,

$T_A - T_B = ma$, where m is the mass of the string, and a is its acceleration upwards (which is zero in this case). Then, as we are assuming that $m = 0$, $T_A = T_B = T_{AB}$(say) (and this also holds for

$a \neq 0$).

By Hooke's law, $T_{AB} = \frac{\lambda_1 x_1}{l_1}$ and $T_{BC} = \frac{\lambda_2 x_2}{l_2}$

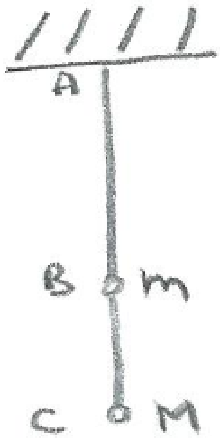
At B, we can apply N3L: the force applied to AB by BC is equal and opposite to the force applied to BC by AB. Thus $T_{AB} = T_{BC}$.

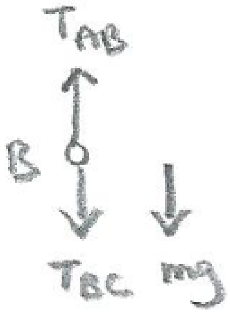
[Alternatively, we can imagine B to be an object of zero mass, and apply N2L to B, giving $T_{AB} - T_{BC} = (0)a$]

At C, we can apply N2L: $T_{BC} - Mg = 0$

So $\frac{\lambda_1 x_1}{l_1} = \frac{\lambda_2 x_2}{l_2} = Mg$.

(2) Suppose now that there is also an object of mass m at B (as shown below). The system is still in equilibrium. The force diagram for B is also shown.





Applying N2L to B, $T_{BC} + mg - T_{AB} = 0$,

and once again, $T_{BC} - Mg = 0$

Hence $T_{AB} = T_{BC} + mg = (M + m)g$