Rolling Wheel – Friction (14 pages; 31/5/24)

See also “Friction” and “Rolling Wheel – Speed of point on circumference”.

Contents

(A) Overview

(1) Friction is normally thought of as a force creating resistance to the motion of an object (eg a block sliding on a surface).

In the case of someone walking on a rough surface, the horizontal component of the reaction of the surface on the person is the frictional force that accelerates the person forward.

This is a case of static friction, which is acting to oppose attempted relative motion of the two surfaces in question (the person’s shoe and the ground).

If the person slows themselves down by digging the front of their shoes into the ground, then the horizontal component of the reaction of the surface on the person now acts to decelerate them.

(2) Car accelerating from rest on level ground – front wheels

Consider a car that is initially at rest on level ground. In the absence of friction, a torque applied to the front wheels would cause them to spin, and the car would not move forward.

Friction on the front wheels opposes attempted relative motion between the wheel and the ground. If the torque is clockwise, as in Diagram 1 below, then the friction acts in the direction of motion of the car. This is static friction, with no relative motion of the two surfaces (the wheel and the ground). In this situation, the wheel starts to roll at an ever faster rate, until either the torque is removed, or the frictional force is overcome (in which case the wheel slips).

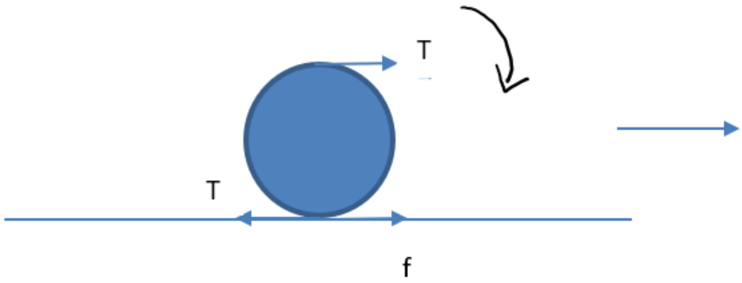

Diagram 1

When a wheel rolls (without any slipping), its point of contact with the surface is stationary (see "Rolling Wheel – Speed of point on circumference"). It is effectively toppling continuously.

(3) Car accelerating from rest on level ground – rear wheels

Were there to be no friction on the back wheels, then these would be dragged along without spinning (by a force F, as in Diagram 2 below).

With friction, attempted dragging is opposed, and so the frictional force f_1 is in the opposite direction to the motion of the car. Once again, the friction is static and the wheel rolls.

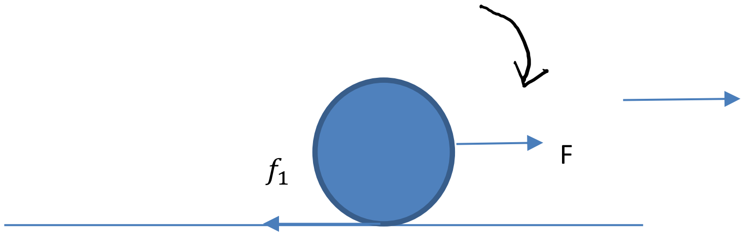

Diagram 2

(4) Car moving at constant speed on level ground

If the torque is removed, then there is no force attempting to produce relative motion between the wheel and the ground. The point of contact remains stationary, and now no frictional force is needed in order to keep it so. In the absence of any resistances to motion, the wheel is now rolling at constant speed.

(Were there to be a frictional force f, as in Diagram 1, but no torque T, then there would be a net translational force f on the wheel, and hence a translational acceleration, contradicting the assumption of constant speed.)

(5) Car moving on a level smooth surface

With the car initially rolling at constant speed, suppose now that the ground were suddenly to be replaced with a completely smooth surface. In the absence of any external forces (apart from gravity and the normal reaction of the ground on the wheel - which have no bearing on this issue - and assuming that there are no resistances to motion), the wheel continues to move with the same constant speed to the right, and with the same rate of rotation, by Newton's 1st law. Apart from the need to support the wheel against gravity, it is as though the surface is no longer there. Technically the wheel is still rolling, because the point of contact is still stationary.

Intuitively this may not seem right, as we tend to think of rolling as involving some interaction between the surfaces. This can lead to the (false) assumption that friction is involved in rolling, even at constant speed. (Once again, were there to be a frictional force, then this would imply translational acceleration.)

(6) Uphill or downhill motion

As will be seen in the Examples, when a wheel is moving up or downhill there will be a component of the car's (etc) weight acting along the slope. Unless exactly countered by an appropriate accelerating or decelerating torque (see Example 6), this will ensure that a frictional force will be present (even when the wheel is not subject to a torque).

(7) Slipping

If rolling is not possible without the limiting static friction ($\mu_S R$) being exceeded, then the wheels will slip. The friction involved will then be dynamic friction.

Note that it is possible for there to be a combination of rolling and slipping. When a ten-pin bowling ball is thrown, for example, it hits the ground at speed, and so the point of contact will not initially be stationary. The ball will start to rotate, at an increasing rate, until the rolling condition is met (see Example (B)(1) for the rolling condition).

(8) Work done by friction

No work is done by static friction, because there is no displacement at the point of contact (which is stationary).

Thus, even when the frictional force is in the opposite direction to motion, no work has to be done against this friction, provided that the wheel is rolling. (If slipping occurs, then work is done against friction.)

Friction does however have the effect of converting rotational kinetic energy into translational kinetic energy.

(9) Rolling friction

In practice a wheel will be deformed near the point of contact with the surface and, because of prolonged contact with the surface, negative work will be done by so-called 'rolling friction', which is therefore a type of resistance to motion.

There will also be friction at the axle of the wheel. (This will do negative work because one surface will be moving over another.)

The Examples are only concerned with friction at the point of contact between the wheel and the surface - resistances to motion will be ignored.

(B) Examples

(1) Accelerating via a torque on level ground

(2) Wheel pulled on level ground

(3) Braking via a torque on level ground

(4) Rolling uphill

(5) Rolling downhill

(6) Braking via a torque uphill

(7) Accelerating via a torque downhill

These Examples concern a wheel of a car of mass λM (with $0 < \lambda < 1$) and radius r, moving to the right, on rough ground; M being the mass of the car that is borne by the wheel in question.

(1) Accelerating via a torque on level ground

(also applies to the rear wheel of a bicycle being pedalled)

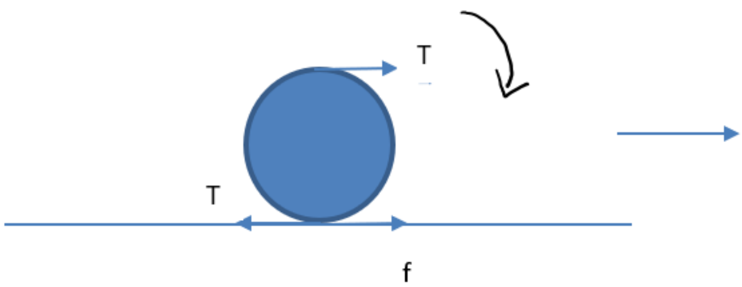

Diagram 1 (repeated)

[Only forces affecting the motion are shown in the diagram.]

Equations can be set up as follows:

N2L applied to translational motion of the wheel:

$T - T + f = M\dot{v}$ (E1),

where v is the translational velocity

[Note that the situation is not the same as that of a block having a force T (to the left) applied to it. The wheel has to be considered as a whole, with a couple being applied to it, as well as the frictional force.]

Net moment of forces about centre of mass = Moment of inertia (about centre of mass) × Angular acceleration:

$\tau - rf = I_W\dot{\omega}$ (where torque $\tau = 2rT$) (E2),

where $\dot{\omega}$ is the angular acceleration (taking clockwise as the positive direction) and $I_W = \frac{1}{2}(\lambda M)r^2$

Rolling condition: the point of contact of the wheel with the ground is stationary. The translational velocity of the point of contact has two components: v for the centre of mass, and $-\omega r$ due to rotation about the centre of mass; so that $v - \omega r = 0$, and differentiating: $\dot{v} = \dot{\omega}r$

From (E1), $f = M\dot{v}$;

then, from (E2), $\tau - rf = I_W\dot{\omega}$

$\Rightarrow \tau - rM\dot{v} = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$

$\Rightarrow \tau = (1 + \frac{\lambda}{2})Mr\dot{v}$

$\Rightarrow \dot{v} = \frac{\tau}{(1+\frac{\lambda}{2})Mr}$

When the wheel is on the point of slipping, $f = \mu_s Mg$ (where μ_s is the static coefficient of friction).

Then, from (E1), $f = M\dot{v} \Rightarrow \mu_s Mg = M\dot{v} \Rightarrow \dot{v} = \mu_s g$ (ie this is the maximum acceleration possible).

(2) Wheel pulled/pushed on level ground

(Applies to a rear wheel of a car (pulled via the chassis); also the front wheel of a bicycle that is being pedalled.)

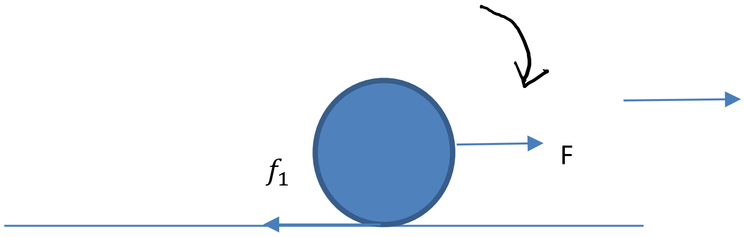

Diagram 2 (repeated)

The wheel is pulled/pushed by the force F, and friction f_1 now acts to the left, to oppose the attempted motion.

Now the equations are:

$F - f_1 = M\dot{v}$

$f_1 r = I_W \dot{\omega}$

$\dot{v} = \dot{\omega} r$

(3) Braking via a torque on level ground

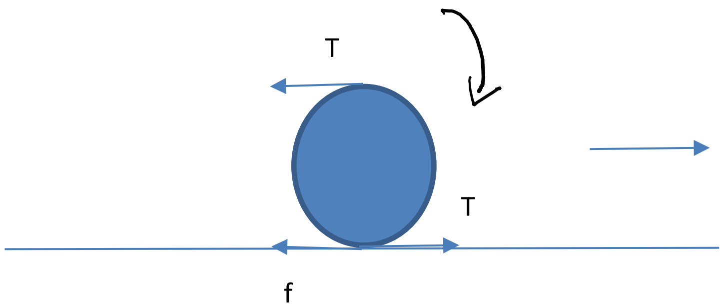

Diagram 3

The diagram shows a wheel subject to an anti-clockwise torque, but still rotating in a clockwise sense, and moving to the right.

The torque is attempting to cause the wheel to slide (to the right) at the point of contact, and so a frictional force will now act to the left.

The equations of motion are:

$-T - f + T = M\dot{v}$ (where the acceleration $\dot{v}$ is negative)

$-\tau + rf = I_W\dot{\omega}$ (where $\tau = 2rT$)

$\dot{v} = \dot{\omega}r$

When the wheel is on the point of slipping,

$-f = M\dot{v} \Rightarrow -\mu_s Mg = M\dot{v} \Rightarrow \dot{v} = -\mu_s g$

So $\mu_s g$ is the maximum deceleration possible; and when the wheel starts to slides, so that $f = \mu_d Mg$ (where μ_d is the dynamic coefficient of friction), the maximum deceleration is

$-\dot{v} = \frac{f}{M} = \mu_d g$, which is usually a lower value than $\mu_s g$. Thus if slipping occurs whilst braking, the stopping distance is greater.

(4) Rolling uphill

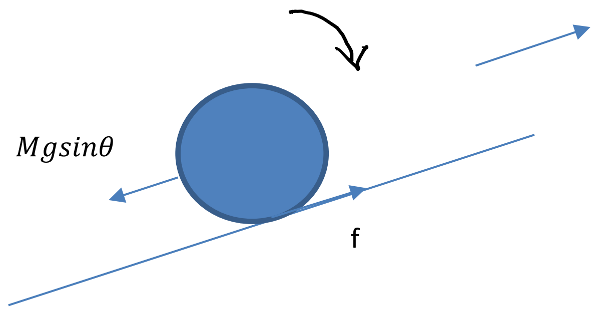

Diagram 4

[This is equivalent to the wheel of a bike that is being pulled by the handlebars, so that it decelerates (the pulling force taking the place of the component of the weight, $Mgsin\theta$).]

The component of the weight $Mgsin\theta$ down the slope will tend to cause the stationary point of contact to move, and friction opposes this force.

The equations of motion are:

$f - Mgsin\theta = M\dot{v}$ (where we expect $\dot{v}$ to be negative)

$-rf = I_W\dot{\omega}$

$\dot{v} = \dot{\omega}r$

So $-r(Mgsin\theta + M\dot{v}) = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$;

$-gsin\theta - \dot{v} = \frac{1}{2}\lambda\dot{v}$

and so $\dot{v} = -\frac{gsin\theta}{(1+\frac{\lambda}{2})}$

(5) Rolling downhill

[This is mathematically equivalent to the wheel of a bike that is being pushed by the handlebars, so that it accelerates - with the component of the weight down the slope taking the place of the pushing force.]

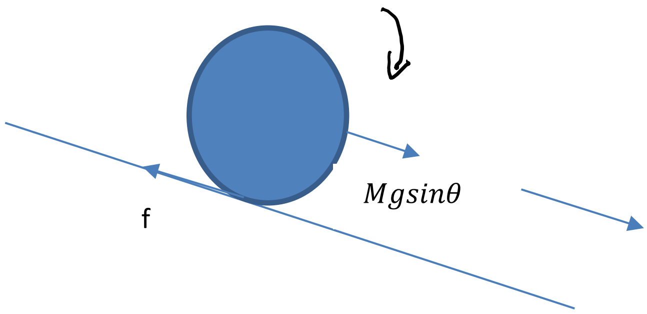

Diagram 5 (where θ is the angle of the slope)

There is a force $Mgsin\theta$ down the slope, attempting to cause the wheel to slide, and friction opposes this force.

The equations of motion are:

$$-f + Mgsin\theta = M\dot{v}$$

$$rf = I_W\dot{\omega}$$

$$\dot{v} = \dot{\omega}r$$

So $r(Mgsin\theta - M\dot{v}) = \frac{1}{2}(\lambda M)r^2\frac{\dot{v}}{r}$;

$$gsin\theta - \dot{v} = \frac{\lambda}{2}\dot{v}$$

and so $\dot{v} = \frac{gsin\theta}{(1+\frac{\lambda}{2})}$

(6) Braking via a torque uphill

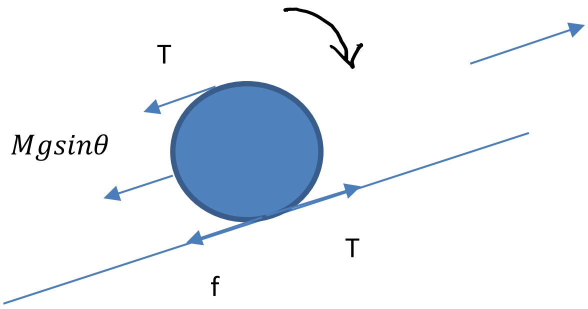

Diagram 6

The direction of friction will be seen to depend on the torque.

Suppose for the moment that the frictional force acts down the slope:

The equations of motion are:

$-T - Mgsin\theta - f + T = M\dot{v}$

$-\tau + rf = I_W\dot{\omega}$ (where $\tau = 2rT$)

$\dot{v} = \dot{\omega}r$

So $-\tau + r(-M\dot{v} - Mgsin\theta) = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$;

$-\tau - Mrgsin\theta = (1 + \frac{\lambda}{2})Mr\dot{v}$

and $\dot{v} = -\frac{\tau}{\left(1+\frac{\lambda}{2}\right)Mr} - \frac{gsin\theta}{(1+\frac{\lambda}{2})}$

Now, $f = -M\dot{v} - Mgsin\theta$,

and, as the frictional force is supposed to act down the slope,

$f > 0$, so that $\dot{v} < -gsin\theta$;

ie $-\frac{\tau}{\left(1+\frac{\lambda}{2}\right)Mr} - \frac{gsin\theta}{(1+\frac{\lambda}{2})} < -gsin\theta$,

or $\frac{\tau}{(1+\frac{\lambda}{2})Mr} > (1 - \frac{1}{\left(1+\frac{\lambda}{2}\right)})gsin\theta$;

ie $\tau > Mr\left(\left(1+\frac{\lambda}{2}\right)-1\right)gsin\theta = \frac{\lambda}{2}Mrgsin\theta$

The frictional force acts up the slope instead if $\tau < \frac{\lambda}{2}Mrgsin\theta$

(If $\tau = \frac{\lambda}{2}Mrgsin\theta$ then there is no friction.)

(Consider the extreme case where $\tau = 0$. This is Example (4): Rolling uphill, where friction acts up the slope.)

(7) Accelerating via a torque downhill

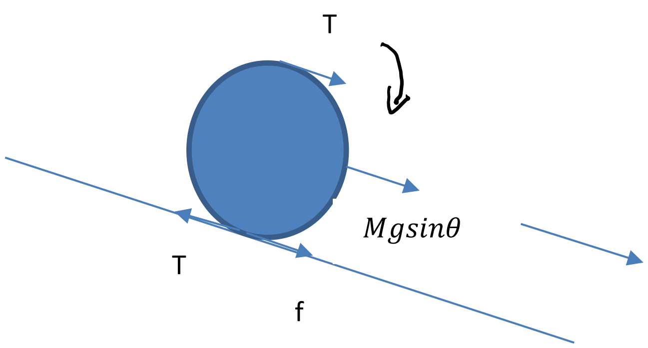

Diagram 7

Suppose for the moment that the frictional force acts down the slope:

The equations of motion are:

$T + Mgsin\theta - T + f = M\dot{v}$

$\tau - rf = I_W\dot{\omega}$ (where $\tau = 2rT$)

$\dot{v} = \dot{\omega}r$

So $\tau - r(M\dot{v} - Mgsin\theta) = \frac{1}{2}\lambda Mr^2\frac{\dot{v}}{r}$;

$\tau + Mrgsin\theta = (1 + \frac{\lambda}{2})Mr\dot{v}$

and $\dot{v} = \frac{\tau}{(1+\frac{\lambda}{2})Mr} + \frac{1}{(1+\frac{\lambda}{2})}gsin\theta$

Now, $f = M\dot{v} - Mgsin\theta$, and, as the frictional force is supposed to act down the slope, $f > 0$, so that $\dot{v} > gsin\theta$;

ie when $\frac{\tau}{(1+\frac{\lambda}{2})Mr} + \frac{1}{(1+\frac{\lambda}{2})}gsin\theta > gsin\theta$,

or $\frac{\tau}{(1+\frac{\lambda}{2})Mr} > (1 - \frac{1}{\left(1+\frac{\lambda}{2}\right)})gsin\theta$;

ie $\tau > \left(\left(1 + \frac{\lambda}{2}\right) - 1\right)Mrgsin\theta = \frac{\lambda}{2}Mrgsin\theta$

The frictional force acts up the slope instead if $\tau < \frac{\lambda}{2}Mrgsin\theta$

(Consider the extreme case where $\tau = 0$. This is (6) Rolling downhill, where friction acts up the slope.)