STEP Exams - Technique (9 pages, 20/7/24)

(A) Complications

(B) Common Pitfalls

(C) Algebra

(D) Presentation of work

(E) Checking

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(A) Complications

A typical pattern is that the first part of a question is straightforward, but a modification introduced into the second part means that a complication has to be taken into account (eg the case where $x = 0$ may have to be treated separately, to avoid a division by zero). The consequence of missing this complication may just be that full marks are not obtained, but it has been known for answers to be rendered worthless when the complication is missed.

Even if a complication cannot be acted on fully, it is probably worth at least showing an awareness of the problem. Borderline candidates (those who have just missed out on the required STEP grades) will have their scripts examined by the tutors of the college (or university) that made them the offer, and they will be looking for things that set them apart from other candidates with similar scores. However, it has been stated by the examiners that marks are only awarded for doing work, not for saying what work would be done were more time available.

A word of warning though: sometimes you may identify an issue that you feel needs addressing, but it turns out that the examiners are not interested in it for some reason (eg if the question is sufficiently demanding in other respects). You may want to save discussion of this issue until the end of the exam (if there is nothing better to do then).

(B) Common Pitfalls

(1) Not using a specified method. Even if a superior method is used instead, zero marks are usually awarded for not using the method requested in the question.

(2) Using ⇒ when ⇔ is required. (See (H)(4) for "if and only if" proofs.)

(3) Losing a solution of an equation by dividing out a factor.

(4) Multiplying an inequality by a quantity without realising that it is (or could be) negative (eg $\ln\left(\frac{1}{2}\right)$).

(5) Not considering all cases.

(6) Spurious solutions

Example: $x = 2 \Rightarrow x^2 = 4 \Rightarrow x = 2 \; or \; x = -2$

"$x = -2$" is termed a 'spurious solution'.

The mathematical meaning of "$x = 2 \; or \; x = -2$" is:

It is the case that at least one of the statements "$x = 2$" and "$x = -2$" is true; ie it doesn't mean that we are free to choose between $x = 2$ and $x = -2$.

[In this example, it is not possible for both $x = 2$ and $x = -2$ to hold at the same time, but we could consider instead $xsinx = 0 \Rightarrow x = 0 \; or \; sinx = 0$]

An acceptable way of dealing with possibly spurious solutions is simply to check that each apparent solution satisfies the original equation (or inequality etc).

(7) Not justifying an argument fully (eg " $a^3 < b^3 \Leftrightarrow a < b$ " probably needs "because $y = x^3$ is an increasing function").

(8) Sometimes a method that is convenient for the first part of a question may not be easily extended to a more complicated second part. So it may be necessary to look ahead in the question.

(C) Algebra

(1) Algebra can feature in many questions (including Mechanics and Probability). Examiners are always bemoaning candidates' shortcomings regarding algebra. The amount of algebraic working required can be quite extensive (in general, much greater than for the MAT paper). At A Level, if the algebra becomes complicated it is usually the case that you have gone wrong somewhere, but this isn't true for STEP.

(2) Provided the number of equations equals the number of unknowns, a unique solution may be possible. Though note that the question may have been designed so that one variable cancels out, in which case a smaller number of equations may be sufficient. Also, if only a ratio of two variables is required, one less equation is needed, and the method of equating coefficients provides a way of obtaining more than one piece of information from a single equation.

In vector questions, each equation will represent 2 or 3 components (similarly for complex numbers), so that it may be worthwhile creating a new unknown if it means generating 2 or 3 equations.

eg if the vectors $\underline{u}$ & $\underline{v}$ are parallel, we can write $\underline{u} = k\ \underline{v}$

(3) Use letters to represent recurring expressions.

(D) Presentation of work

(1) If a result is to be proved, and if you are writing this result out at the start of the question (though this may not be necessary), make it clear that this is something to be proved (rather than part of your working), so that the marker doesn't have to read over it. The abbreviation 'rtp' [result to prove] is likely to be familiar to a STEP marker.

(2) Suppose a candidate has written the following:

$xy < xz$

$y < z$

$x > 0$

As it stands, it is not clear whether $y < z$ is supposed to lead on from $xy < xz$ (which would be incorrect as it stands, as x could be negative), or whether $y < z$ is a result established earlier (or perhaps stated in the question). Likewise, is $x > 0$ being deduced, or brought in from somewhere else?

A revised version of the above might be:

From (*), $xy < xz$

Also, the question states that $y < z$

Hence $x > 0$

(3) Example

Original version

$\frac{dr}{d\theta} = cos\theta - sin\theta$

$\frac{dr}{d\theta} = 0 \Rightarrow tan\theta = 1 \; (cos\theta \neq 0)$

Improved version

$\frac{dr}{d\theta} = cos\theta - sin\theta \quad (*)$

$\frac{dr}{d\theta} = 0 \Rightarrow \; cos\theta = sin\theta$

$\Rightarrow \frac{sin\theta}{cos\theta} = 1$; ie $tan\theta = 1$, provided $cos\theta \neq 0$

But, from (*), $cos\theta = 0 \Rightarrow sin\theta = 1$, so that $\frac{dr}{d\theta} \neq 0$

So, as we are assuming that $\frac{dr}{d\theta} = 0$, it follows that $cos\theta \neq 0$, and hence $tan\theta = 1$

(4) To avoid any uncertainty, each statement in your working needs something to show where it comes from. Often this will just be an implies sign (⇒) (but see the discussion below about the "if and only if" argument).

Introducing something with the word "consider" is a good way to indicate that you are starting a new line of argument.

(5) Note that the ⇒ sign can sometimes be ambiguous.

For example, take the following statement from (3) above:

"$\frac{dr}{d\theta} = 0 \Rightarrow \ cos\theta = sin\theta$". As it stands, this could be interpreted

as either (a) "It has been established that $\frac{dr}{d\theta} = 0$, and so it follows that $cos\theta = sin\theta$",

or as (b) "If it were [hypothetically] the case that $\frac{dr}{d\theta} = 0$, then it would follow that $cos\theta = sin\theta$".

To indicate that (a) is intended, rather than (b), we could write instead:

"As $\frac{dr}{d\theta} = 0$, it follows from $(*)$ that $cos\theta = sin\theta$".

(6) For "if and only if" proofs, it may be acceptable to indicate that the line of reasoning is reversible (assuming that this is the case, of course); ie by use of the ⇔ sign at each stage.

When proving that $A = B$, be careful not to adopt the following (incorrect) argument:

"$A = B \Rightarrow \cdots \Rightarrow Y = Z$ (eg $0 = 0$) [something that is clearly true]"

What we want to show instead is that

"$Y = Z$ [which is clearly true] $\Rightarrow \cdots \Rightarrow A = B$

This can be got round by writing

"$Y = Z \ \Leftrightarrow \cdots \Leftrightarrow A = B$

(though it isn't usually thought to be that elegant (especially when ending with $0 = 0$), and is best avoided if possible).

A more acceptable method is to show that $A = A'$ and that

$B = B'$, and then demonstrate that A' can be rearranged into B'; or alternatively show that $A' - B' = 0$

(7) Explanations

It can be a good idea to explain what you are doing, for the marker's benefit. (However, credit can't be given for an explanation of what you would do, if you had more time.)

(8) Showing working

Reasons for showing plenty of working:

(a) It enables you to easily check over your work (as you complete each line).

(b) The marker is kept happy, by not forcing them to do algebra in their head.

(c) If you make a slip, then your method may still be clear (bearing in mind that method marks are usually available).

(d) Full marks may not be awarded in the case of a 'show that' result if there is a jump to the result.

(E) Checking

(1) Read over each line before moving on to the next one. This is the most efficient way of picking up any errors.

(2) Just before embarking on a solution, re-read the question. Also re-read it when you think you have finished answering the question, in case there is an additional task that you have forgotten about.

(3) It is also a good idea to re-read the question if you find yourself getting bogged down in awkward algebra, or if you don't seem to be getting anywhere (though note that STEP questions are prone to involving lots of awkward algebra!)

(4) Look for ways of checking your answer, or part of your working:

(a) if simultaneous equations have been solved, the solutions may be substituted back into the equations

(b) there may be an alternative approach to (for example) an arithmetic calculation

(c) a reasonableness check

(F) Use of time

(1) Before embarking on a solution, consider how likely it is that it will work, and how much time it will take.

(2) You might like to save a relatively straightforward task to complete in the last few minutes of the exam, rather than frantically looking through the paper for something to check.