Vectors Q26 (3/5/24)

Write the following Cartesian equations of planes in parametric form:

(i) $x + y + z = 1$ (ii) $x + y = 1$ (iii) $x = 1$

Solution

(i) For this case, we can set $x = \lambda$ and $y = \mu$, if a parametric form of this type is possible:

$$\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ b \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ c \end{pmatrix} \quad (*)$$

This means that a will be 1.

Then $\begin{pmatrix} 1 \\ 0 \\ b \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 1 \\ c \end{pmatrix}$ will be valid direction vectors in the plane,

provided that they are perpendicular to the normal to the plane; ie that there are solutions to $\begin{pmatrix} 1 \\ 0 \\ b \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$ and $\begin{pmatrix} 0 \\ 1 \\ c \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$; so $b = c = -1$

Thus a possible parametric form is

$$\underline{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

[Alternatively, $x + y + z = 1 \Rightarrow \lambda + \mu + z = 1$,

so that $z = 1 - \lambda - \mu$, and so $a = 1, b = -1$ & $c = -1$]

(ii) In this case, $\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ b \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ c \end{pmatrix}$

isn't possible, as $x = y = 0$ doesn't satisfy $x + y = 1$

Instead, we can try $x = \lambda$ and $z = \mu$, so that $y = 1 - \lambda$

This gives $\underline{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$,

and, as a check, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ satisfies $x + y = 1$,

and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$ & $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$

(iii) Here, we can obviously set $x = 1, y = \lambda, z = \mu$

Thus a possible parametric form is

$$\underline{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$